THE NATIONAL GEOMETRY SYLLABUS COMMITTEE.

SUB-COMMITTEE ON LOGICAL CONSIDERATIONS.

BY EUGENE R. SMITH.

At the 1908 summer meeting of the National Educational Association the Mathematics Section authorized the appointment of a National Geometry Syllabus Committee, with Dr. H. E. Slaught as chairman. Lack of permanence of organization, and therefore of right to appropriate money, on the part of the section temporarily stopped the work, although Dr. Slaught continued to canvass the country for representative men to form the committee.

At the Baltimore meeting of the American Federation of Teachers of the Mathematical and the Natural Sciences, as the representative of the Middle States and Maryland Association, I presented the matter to the Federation, and urged the continuance of the work apparently dropped by the National Educational Association. The Federation at once adopted the scheme, accepted Dr. Slaught as chairman, and to avoid losing the work already done by him, asked him to nominate to the executive committee the teachers he thought should be on the committee. Soon afterwards the National Educational Association again took up the matter, and as a result, a joint committee of fifteen was appointed. This committee was then subdivided by the chairman into three sub-committees, of which I represent the one asked to report on the logical considerations of geometry. This committee is constituted as follows: David Eugene Smith, chairman; Eugene R. Smith, secretary; William Betz, of Rochester High School; C. L. Bouton, of Harvard, and William Fuller, of Mechanics Arts High School, Boston.

The work expected was outlined as follows: "The consideration of axioms, definitions (including new terms, symbols, distribution, etc.), assumptions, informal proofs; treatment of limits and incommensurables; time and place in the curriculum; purpose; historical notes; and other related topics.

The committee has considered all the points mentioned in the foregoing paragraph, and I will touch briefly on the details of the report that will be made to the general committee.

Axioms and Postulates.

The committee recommends the usual ones, with little modification. It does not approve of the attempt to reduce the assumptions of elementary geometry to a minimum nor does it believe that all the implicit assumptions referring to existence, betweenness, etc., are of advantage. A substitution axiom is recommended on account of its convenience in many proofs.

It may perhaps be of interest if the list of axioms and postulates is given entire.

Axioms.—(1) The equality axioms, including equals plus equals, equals minus equals, equals times equals, equals divided by equals, like powers and like roots of equals, things equal to the same thing are equal to each other. (2) The axioms relating to the use of unequals; including unequals plus and minus equals, times equals, and divided by equals, unequals plus unequals; unequals subtracted from equals; and an axiom showing that if a first quantity is greater than a second, a second than a third, etc., the first is greater than the last. (3) A substitution axiom. (4) The whole is greater than any of its parts, and is equal to the sum of all its parts.

Postulates.—(1) One straight line, and only one can be drawn through two given points. (2) A straight line may be produced to any required length. (3) A straight line is the shortest path between two points. (4) A circle may be described with any given point as a center and any given line segment as a radius. (5) A figure may be moved from one place to another, without altering its size or shape. (6) All straight angles are equal. (7) Through a given point one line and only one can be drawn parallel to a given line.

Terms.

The only new term directly recommended is "congruent," and this is now so well established as to be old rather than new. The terms "scholium" and "mixed line" are noted as obsolete and useless; the new terms "ray," "sect" and "transverse angles" are spoken of as not yet in common use, and teachers are left free to use them or not as they choose, with the idea that they will make their way if they are worthy of use. The two uses of the word "circle" are compared; the newer use of

"circle," meaning the line rather than the surface bounded, is recognized, but the committee takes the position that it is not as important that consistent use be made of either method of defining throughout geometry, as that the terms be so defined that the meaning is clear to the pupils.

Symbols.

Those in common use, including the ordinary algebraic signs, and the usual signs for the geometric figures are the only ones needed. New symbols should not be used unless they receive the sanction of the mathematical world. There is some question of a sign for "congruent," but the usage is so varied that the committee is not ready to recommend the adoption of any one. If I may express a personal opinion, I believe that the combination of the equal and the similar signs is the logical one for final adoption. The equal sign alone is too confusing on account of its various meanings, and the identity sign has a somewhat different significance, while, on the other hand, the combination of the equal and similar signs is used for nothing else, and expresses the double truth about congruence.

Definitions.

The simpler concepts are decided to gain nothing by the attempt to define them. Such concepts are point, line, surface, angle, space, etc.; this does not mean that such concepts should not be discussed as to their definite usage, but simply that, as there are no simpler terms by which to define them, pupils should not be asked to memorize formal definitions.

The easily defined terms that are used as a basis for propositions should be defined accurately, although the exact form of the definition is not material.

It is recommended that definitions that are not required until later parts of the geometry course be given when needed, although there is no objection to their being grouped in the text for ready reference.

Informal Proofs.

The committee recognizes the fact that many geometric truths are most easily taught without a formally worded proof. Many of the most evident facts follow almost directly from the defi-

nitions, and a word of explanation is all that is necessary. In such cases the understanding of the pupil is helped rather than hindered by the simplicity of the explanation, while the rigor of the development is not dangerously weakened. A few of these statements are:

If one straight line meets another the sum of the two adjacent angles is a straight angle, and conversely (and related propositions).

All straight angles are equal (if not used as a postulate).

Two straight lines can intersect in but one point.

If two angles are unequal, the greater angle has the less complement (and related propositions).

A straight line can cut a circle (circumference) in two points only.

Circles of equal radii are equal, and related statements.

A straight line can have but one point of bisection, and the related case for angles.

The bisectors of vertical angles lie in one straight line.

The bisectors of adjacent supplemental angles are perpendicular to each other.

All radii (and all diameters) of the same circle are equal.

A circle can have but one center.

Propositions relating to the conditions under which two circles (circumferences) intersect.

Polygons similar to the same polygon are similar to each other.

These propositions are, of course, only types of the ones to be proved informally; there are many others of a similar nature.

The committee believes that the experience of teachers both in this country, and even more in some of the European countries where this method is quite freely used, has shown that good results follow it when it is used in moderation. That it should be carried to an extreme would undoubtedly be dangerous and defeat its own purpose.

Limits and Incommensurables.

The committee takes the position that schools should not be required to teach this part of the subject, in other words, that it should not be considered a part of college preparation. On

the other hand, some explanation of the necessity for incommensurable case proofs is necessary and each teacher is left free to give to his pupils whatever they can assimilate, with the understanding that what is taught is for the purpose of giving a clearer grasp of the subject, and not as part of the required course. Many of the best schools have given up teaching this part of the subject some time ago, and it seems that this recommendation is only the formal statement of what is already becoming an accomplished condition.

Time and Place in the Curriculum.

For the ordinary secondary school, that is, high school or preparatory school, eight years of elementary school are presupposed. In the first year of the secondary school course algebra is usually taught, and the position of the committee is that geometry, except where a teacher believes in starting it with algebra and teaching them together, should follow in the second school year, and should have one and a half years for its completion. This means one and a half school years of five periods a week, the periods being the ordinary ones of approximately forty minutes. In this time it is desirable, if possible, to give the pupils some idea of the first part of solid geometry. This amount of time is perhaps more than is usually allowed, but it is none too much for a subject of the importance of geometry, and the adoption of this recommendation would do a great deal toward enabling teachers to get all the good possible out of the geometry course.

Purpose of Teaching Geometry.

There has recently been some attempt on the part of certain teachers to make the practical side of geometry dominate its teaching. This attempt is not a new thing, for similar movements have originated at various periods of its history, and with no important result. That genuine applications of any subject have a decided value goes without saying, but that the search for applications of, at least, doubtful character should carry teachers away from the real purpose of their teaching is a dangerous tendency. The committee welcomes anything that adds interest to the subject or broadens its outlook, but it feels

strongly that the chief value of the study of geometry arises from the fact that it is an exercise in logic, and logic of a kind concrete enough to be understood by the immature mind of a secondary school student. It therefore takes the position that the chief purpose of geometry is cultural, and deprecates any attempt to emasculate the subject in the search for so-called "utilitarian" applications.

The question as to whether a short course in practical geomtery would be wise in certain kinds of schools did not seem to come in the province of this committee. The committee would probably agree that for certain classes of pupils such a course would be wise. The position taken refers to the ordinary secondary school where the pupil presumably desires as broad a education as the time at his disposal allows.

Historical Notes.

The committee calls attention to the fact that historical notes about the various interesting features of the geometry add value and interest to the course. There are many sources from which such material can be obtained, and here again anything of a broadening character should be welcomed.

Solid Geometry.

The general principles already spoken of apply equally to solid geometry and little more need be said. The committee states the axioms and postulates that seem necessary, and suggests that latitude be allowed in the use of terms such as prismatic space and others in more or less use, while it does not specifically add any to the commonly used terms. The position taken in regard to limits and incommensurables holds also for solid geometry.

In regard to the purpose of the study of solid geometry, two are added to the one relating to the cultural side: to present a reasonable range of applications to mensuration; to cultivate the power of visualization, more especially of solid figures from two dimensional drawings.

As regards the first, it is of course evident that while plane geometry offers a comparatively small proportion of applicable propositions, solid geometry is used for many important meas-

urements, and so may fairly be said to have its applications to mensuration as an integral part of the purpose for which it is taught.

The cultivation of the power of visualization has probably not been given the recognition it deserves; and there can be no doubt that solid geometry possesses to a high degree the qualifications necessary to training this ability.

In conclusion I wish to say that this committee has tried to avoid any radical recommendations such as have not yet been approved by the mathematical world, while it has meant to be open minded toward improvements in geometry teaching. Whether it has succeeded in its aim is for teachers to judge from its report, but if the report is accepted as a sane and helpful one, then every teacher should work for its adoption as a standard, in spite of any personal preferences that may hinder his agreeing with all its details. It would be absolutely impossible for all teachers of geometry to agree as to all the details in its teaching; and it is fortunate that it is so, for otherwise there would be little progress, but it is possible for us to be ready to coöperate with each other in setting up a standard of uniformity. Even the members of this committee do not absolutely agree on all the details of its report, and this report is, as would of course be necessary, a consensus of opinion. If it shall appear to represent the consensus of opinion of all teachers of geometry, that for which the committee was appointed will have been accomplished. In making this plea for uniformity, I am thinking of the report of the entire committee as well as of the part that I represent, for perhaps the part in which uniformity will be of most value is the list of theorems being prepared by the second subcommittee.

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